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Negative Exponential function: its versatility in aeronautics

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The named Probability Distribution Function Negative Exponential (e^{-x})) draws attention for its simplicity and versatility. we already talked about it, but there's always something more to say. It appears early in the life cycle of an aircraft (Conceptual Design) and remains in this cycle in various activities until the disposal of the aircraft.

When we substitute x for λt, we have our familiar probability distribution function called Reliability, represented by the letter R, that is:

$$\mathbf{R = e^{-\lambda t}} \qquad (1)$$

where λ is a constant, our equally familiar failure rate given in failures/hour.

Everything in this function is widely usable. The inverse of its parameter λ is another important parameter, the popular MTBF (Mean Time Between Failure), given in hours/failure..

The function is so versatile that it can even generate another extremely important function in Safety Assessment, as we will show later.

As we know, in practice Reliability is the probability of success of an event in a given time interval, in a given flight configuration and under certain conditions.

Among other reasons, the negative exponential function is very special, because a property that only it has. This is called the forgetfulness property, i.e. the system which follows this probability distribution function, after being turned off and on again, "forgets" that already operated before, resuming its operation as never he had operated. A failure is totally random, that is, it is not the a result of deterioration of material (Ref. 1).

In fact, that's how behaves a purely electric or electronics system, today, in most of its operational phase, maintaining an approximately constant failure rate. This is quite different to the mechanical systems, which have a variable failure rate, starting high, as in the electronic system, then passes through a minimum and starts to grow again, starting the peculiar wear of mechanical systems.

To every probability distribution function corresponds a cumulative probability distribution function (CDF-Cumulative Distribution Function).

The CDF corresponding to the reliability is called Unreliability (U) or Fallibility (F).

R and F are complementary and mutually exclusive functions. Therefore, they obey the relation:

$$\mathbf{R + F = 1} \qquad (2)$$

This expression seems obvious for us because the probability of not failing (R) or the probability of fail (F) is 1 (100%), because the event occurs or doesn't occur.

It follows that

$$\mathbf{F = 1 - R = 1 - e^{-\lambda t}} \qquad (3)$$

Now, let's show the versatility of the negative exponential in the generation of a new extremely important function in Safety Assessment.
At a certain moment, a genius envisioned that, in practice, he could develop the expression (1) in

an infinite series of other genius, the mathematician Taylor, given by:

$\mathbf{e^x = 1} + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots,$

with $-\infty < \mathbf{x} < \infty$.

If $x = -\lambda t$, we can write:

$$\mathbf{R = e^{-\lambda t} = 1 +} \frac{(-\lambda t)}{1!} + \frac{(-\lambda t)^2}{2!} + \frac{(-\lambda t)^3}{3!} + \cdots =$$

$$= 1 - \lambda t + \frac{(\lambda t)^2}{2} - \frac{(\lambda t)^3}{6} + \cdots \qquad (4)$$

So, he realized that for $\lambda t < 0.1$, he could be consider as a good approximation only the first two terms in the case of purely electrical and electronic systems; that is:

$\mathbf{R = e^{-\lambda t} = 1 - \lambda t}$. Using (3), we obtain: $F = 1 - (1 - \lambda t)$. Therefore,

$$\boxed{\mathbf{F = \lambda t}} \quad \text{with } \lambda < 0{,}1 \qquad (5)$$

and ready: we have a new function, which is the equation of a line, an expression much easier to work with the parameter λ out of the exponent of an exponential becoming an angular coefficient of a line. That versatility!

Well, the expression (5) – Fallibility - is used in the Safety Assessment quite closely for electrical and electronic systems because the failure rate of these systems is very nearly constant, during the operational life of the system. However, in Safety Assessment, the function is also applied to electromechanical systems_ mechanical, pneumatic, hydraulic (Ref. 2).

Regarding to Reliability (1), it can be used in assessing the probability of success of a mission, in military aviation, and a huge employment, either in military aviation and civil, in the development of logistic technical support, through the parameter λ or its reciprocal, the MTBF, that is:

$$\mathbf{MTBF = 1/\lambda} \qquad (6)$$

The MTBF is often used in logistical factors maintenance and spare parts. The parameter corresponds to the time interval corresponding to 37% of the probability of not fail (reliability), or 67% of the probability of fail (Fallibility). The MTBF is only applicable to repairable systems. If the system is not repairable, i.e., if it is disposable, we must use the Mean Time to Failure MTTF (Ref. 3).

Note that if λ is constant, its reciprocal, the MTBF, is constant too. Thus, strictly, the MTBF would apply only to electronic repairable systems.

The negative exponential, through its parameter (or MTBF), is present in the development of the maintenance scheduled plan (preventive maintenance) of the aircraft through the tool Reliability Centered Maintenance.

In closing, here it is a question with the answer: why do not use the reliability in Safety Assessment, instead of Fallibility? The answer is that the reliability of electronic systems today is very high (R = 0, 99999 ...), which makes it much more sensitive to rounding errors that Fallibility

We stop here our flash. The reader will have a lot more information by consulting the following bibliography.

See you

References:

(1) CLARKE, A. Bruce, DISNEY. Ralph L. Probability and Random Processes for Engineers and Scientists. John Wiley & Sons, Inc., New York (USA), 1970.

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