- Refinement of Failure Rate: Confidence Intervals -

Berquó, Jolan Eduardo – Electronic Eng. (ITA)
Aerospace Product Certifier (DCTA/IFI)
Government Representative for Quality Assurance – RGQ (DCTA/IFI)
jberquo@dcabr.org.br

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This flash is a complement to the MSC 14, in which we have had the opportunity to present a way to estimate the failure rate (λ) of the negative exponential distribution function, by means of tests of life (Life Testing). The question that we put now is: how far can we trust in the values found for failure rate obtained in those tests? This is the reply that we intend to present in this MSC.

We had so the opportunity to see that the value of the rate varies, depending on the type of test, i.e. with or not the replacement of the failed items and if it is terminated after a certain time (finished by the time – Type I) or after a certain number n of failures (test completed by number of failures – Type II).

This difference between values is influenced by several factors, the main one being the amount of items in the sample tested. Of course, if we could perform a test with a sample of thousands of items, this difference would tend to zero, that is, the failure rate obtained by either method would be about the same and very close to the reality.

However, a testing with a large number of items in the sample is almost always economically prohibitive, especially when it comes to equipment, even though simple.

Thus, when we conducted a life testing and reach a value of failure rate, we need to have a certain level of confidence in that estimate.

For example, we could know the answer to the following question: What would be the range of the failure rate with a confidence level of 90% or 95%.

Several researchers have sought to answer this question. Let us consider here the Epstein method (Ref. 3). This researcher was able to demonstrate that if the time to fail is exponentially distributed with a rate λ, then the expression $2n\lambda/\bar{\lambda} = 2\lambda T$ has a "chi-squared" distribution (symbolized by $\chi 2$), with 2n degrees of freedom for the type II test and 2n + 2 degrees of freedom for the test type I.

Remember that

n = number of failed items;

T = Time Accumulated in the Test;

λ = Actual Failure Rate; and

$\bar{\lambda}$= Estimated failure rate in the test.

With the estimated failure rate $\bar{\lambda}$, obtained in the life testing, and with the data of the test (number of items in the sample, number of failures, test time and cumulated time by the items during the test), Epstein has showed that one can calculate the probability of having an actual failure rate λ within a certain interval. This interval was then called "confidence interval" (CI) and their likelihood of being in that range was named "level of confidence" (LC) for the interval.

After some calculations, Epstin determined the expression (1), to calculate the probability of a confidence interval for λ with a certain level of confidence.

$$\Pr\left[\frac{\chi^2_{(\frac{\alpha}{2})}(2n)}{2T} \leq \lambda \leq \frac{\chi^2_{(1-\frac{\alpha}{2})}(2n)}{2T}\right] = 1 - \alpha$$

(1).

Where 1-α is the confidence level (CL), and α is the complement of CL. So, if the confidence level is 0.9 (or 90%), α = 0.1 (or 10%), and α is the probability of the failure rate not be contained in the confidence interval.

Another alternative is the called one-sided Confidence Interval, that is, a interval with the lower end null. The expression would be then::

$$\Pr\left[0 \leq \lambda \leq \frac{\chi^2_{(1-\alpha)}(2n)}{2T}\right] = 1 - \alpha \qquad (2)$$

In our opinion, the expression (2) is more than enough for the vast majority of cases.

Let us go then to the example presented in the MSC 14 for the Test Type I, that is, with replacement of the failed items and finished by time.

Ten resistors have been tested, and eight of these devices failed before reaching 900 hours.

The accumulated time in test for all items that participated in the test was then T = 10 x 900 = 9000h (see MSC 14), and the resulting estimated failure rate was $\bar{\lambda}$= 8/9000 = 8.9 x $10^{-4}h^{-1}$. Let's see if this failure rate is within the confidence intervals with NC (probability) of 90%, 95%, 97.5 and 99%, considering the one-sided confidence interval.

Solution

Note that $\chi^2_{(1-\alpha)}(2n)$ = $\chi^2_{(0.9)}(2.8)$ = $\chi^2_{(0.9)}(16)$. Going to the table, we find on line 16 and column 90 the value 23.54. Thus, we have:

23.54/2T = 23.54/18000⇒ $0 \leq \lambda \leq 1.3 \times 10^{-3}h^{-1}$.

Let us pass to the confidence level of 95% (α=0.5).

We have, for $\chi^2_{(0,95)}(16)$, the value 26.30, resulting the interval $0 \leq 1.5 \times 10^{-3}h^{-1}$.

With 97.5%, we get: $0 \leq \lambda \leq 1.5 \times 10^{-3}h^{-1.}$

With 99%, we would have: $0 \leq 1.8 \times 10^{-3}h^{-1}$.

Now compare the estimated value we have obtained, that is: 8.9 x 10-4h-1 or 0.89 x 10-3h-1.

We can see in the table that the CL for the estimated value is slightly higher than 50%. Therefore, it is a low CL. The value of NC to be adopted is a matter of decision. If we stay with the CL 99%, we would adopt the interval $0 \leq \lambda \leq$ 1.8 x 10-3h-1 and we would consider the upper end as value forλ, that is

$$\lambda = 1.8 \times 10^{-3}h^{-1}.$$

By comparing this value with the estimated rate, we note that $\lambda/\bar{\lambda} \cong 2.1$, i.e. λ is slightly larger than the double of $\bar{\lambda}$. Therefore, it might be prudent to adopt the value of λ and abandon the estimated value$\bar{\lambda}$.

***Conclusion**: when we make a life test, to determine the failure rate, we must complement the process with a Confidence Level (CL) check.*

Thank you

See you

References:

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(3) EPSTEIN, B. **Estimation from Life Test Data.** Technometries. USA, 1960.