Reliability: Tests of Launching Missiles -

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When a company develops a system, it has to do tests, including the so-called functional tests, to ensure that the system meets the design specifications, which arise from the creativity of the company that has designed it and or from the customer requirements, such as those from *Comando da Aeronáutica* (Aeronautics Command) in its acquisitions, or from a civil authority, like ANAC, our Authority in the area of civil aviation.

And tests require a lot of money, being one of the strongest components of the cost of system development.

We know this very well, when we work with aircraft and its equipment.

However, when it comes to missiles, the speech is another. In testing of these systems, we have just a way to do it, that is, we have to launch them to verify if they accomplish the mission, according to the degree of accuracy expected. In general, we have to spend lots of money.

In General, several prototypes are tested, and the design is refined at each attempt with successes and errors, even if we have a design in theory very well substantiated.

Before performing such tests, it is imperative to have a certain amount of predictive successes and failures, which is discussed between customer and the company. The question of time and the costs is carefully discussed between both.

The Program Manager of this system developing, on the client side, needs to have this information, since he will always be updating the authorities who put him in this role.

But to have this prediction, we have to have one theoretical important datum, that is, the probability of missile fulfill its mission in every launch, calculated as well as possible.

From this point, we can have a probabilistic forecast of successes and failures in test launches, in a universe of **n** launches, with **n** established by agreement between customer and the company.

It is exactly about that moment that we refer in this MSC, i.e. we will try to show how to calculate the probability of successes and failures in n launches with the probability of success determined theoretically for each launch.

We have already had the opportunity to participate in discussions about these tests. Thus, we present the following as we treat this subject, when we worked as coordinator of certification systems in DCTA/IFI.

Now, hold on a little of mathematical theory.

The missile tests can be treated as discrete events, i.e. in each launch there are two possible outcomes: success and failure.

The probability distribution more convenient to treat this type of event is a discrete distribution called "Binomial Probability Distribution". That's because these tests meet the necessary and sufficient conditions for the employment of this distribution, ie:

- each launch is independent of the previous launches (the occurrence of previous results does not influence the outcome of the next release);
- each outcome is not known a priori (random outcomes); and
- The probability of success in each launch is the same (constant).

Consider then a series of **n** independent launches in the testing program.

Since the number of launches is a non-negative integer, the set of the possible outcomes, called the Sample Space is given by:

$$S = \{0, 1, 2, 3, \dots, n\} \qquad (1)$$

The binomial distribution is defined by the expression:

$F(x) = Pr[X=x] = \binom{n}{x} p^x (1-p)^{n-x}$, com x = 0, 1, 2, 3, ... , n.

Or

$$\boxed{Pr[X=x] = \frac{n!}{x!(n-x)!}\, p^x . q^{(n-x)}} \qquad (2)$$

Where **x** is the exact expected number of successes in n launches and **p** is the probability of success on each launch, and **q** is the probability of failure on each launch. Therefore, q = 1-p.

The expression $\binom{n}{x} = \frac{n!}{x!(n-x)!}$ (3)

gives us the number of possible combinations of **x** events (successes) in **n** launches. Because they are combinations, we have to remember that (1, 2, 3) is the same as (2, 1, 3), (1, 3, 2), etc.

If we want, for example, to know what is the probability of having exactly 3 successes in 5 launches, we can write:

$F(3) = Pr[X=3] = \frac{5!}{3!(5-3)!} p^3.q^2 = 1{,}46 \times 10^{-2}$.

From the probability theory, we know that the probability of having one or another outcome of all possible outcomes is 1, ie, the sum of the probability of all possible outcomes is 1. This seems clear because surely one of the results always will happen.

Then, the probability of having any outcome of the sample space S is 1.

So, we can write:

$Pr[S] = \sum_{x=0}^{n} Pr[x] = Pr[0] + Pr[1] + Pr[2] + ... + Pr[n] = 1.$ (4)

This expression can be represented by the so-called Binomial of Newton. Namely:

$(p+q)^n = C_1.p^n + C_2.p^{n-1}q + C_3.p^{n-2}q^2 + C_4.p^{n-3}q^3 ... C_n.p.q^{(n-1)} + C_{n+1}.q^n = 1.$ (5)

With the coefficients calculated by expression (3).

This calculation shows that C1 = C2 = 1, regardless of **n**.

Suppose that we will carry out a campaign of 4 launches.

In this case, we have:

$(p+q) = p^4+4p^3q+6p^2q^2+4pq^3+q^4$.

Be p = 0.9 (set theorycally).

What would be the probability of having exactly 3 successes?

We have: P (X = 3) = 4p3q2 = 2.92×10^{-2}.

And if we wanted to have up to 2 successes, that is, 0, 1 and 2 successes?

In this case we would have a cumulative distribution, ie:

$Pr[X\leq2] = Pr[0]+Pr[1]+Pr[2] = 1 - \{Pr[3] + Pr[4] + Pr[5] + Pr[6]\}$.

We have: $Pr[X\leq2] = 1 -(p^6+6p^5q+15p^4q^2) = 9.83 \times 10^{-1}$.

Data such as these will be subject of discussion between customer and the company.

There are several points in these discussions. One, for example, is the issue of calculation of the reliability of the mission. It takes very careful with this calculation because this value is of fundamental importance in the use of the presented expressions. Values far removed from reality can disable one testing campaign.

We would like to add here that the coefficients of the binomial of Newton can be obtained directly from Pascal's Triangle, as shown in Figure 1.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
....

Fig. 1 –Pascal's Triangle

The second number of each line gives the value of **n**.

If, for example, n = 6, we have, from the seventh line:

$(p+q)^6 = p^6 + 6p^5q + 15p^4q^2 + 20p^3q^3 + 15p^2q^4 + 6pq^5 + q^6 = 1.$

Well, we stop at this point. Thanks.

See you

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