- Reliability: Negative Exponential Function - Practical Determination of Failure Rate -

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We know that the negative exponential reliability function is dedicated to electrical and electronic equipment and complex systems like an aircraft, which, with great approximation may be considered a black box (electronics) with wings, control surfaces and engine (and this also with various electronic accessories).

As we know, this function is given by:

$$\mathbf{R = e^{-\lambda t}} \qquad (1)$$

Where λ is a constant called "Failure Rate", and **t** is the average time of the mission. ⍰ The inverse of the constant is called "Mean Time Between Failures" (Mean Time Between Failures), popularly known by the acronym **MTBF**. This parameter is used in maintenance, assuming that the repaired equipment comes back to operation as a new equipment, a condition approximately true since it has high-quality maintenance.

When we refer to non-repairable equipment, the parameter used is the "Mean Time to Fail", known by the acronym **MTTF**. It is used, for example, for lamps.

We can not forget that we just can speak of a constant failure rate after the item has a design mature, a consolidated design, that is, already has a finalized configuration which has passed by a battery of tests, as occurs in its operational phase.

There are theoretical methodologies to estimate λ, from data collected in laboratory testing and field.

We will present here, as always rapidly, some ways to estimate this parameter from practical data. But we must be alert to the fact that this methodology is valid only if we know that the behavior of the item fits the exponential distribution.

A methodology that allows us to estimate λ and **MTTF** are the called Life Testing.

In life tests, a sample of the same type of items is subjected to tests <u>in an environment similar to that in which the items must operate</u>, and the instants at which failure occurs are recorded.

In general we use two types of tests:

a) with replacement of the failed items; and
b) without replacement of the failed items.

When **n** items are submitted to life tests with or without replacement of the failed item, it is sometimes necessary to stop the test in a certain point due to the long lifetime of the item, and so perform the reliability analysis based on the data obtained so far.

Tests that are stopped before there is the failure of all the sample items are called "censored " or "truncated”. We will not enter here into the details of the nomenclature used for censored , as right censored, left censored, etc. The reader can himself deepen into these details, referring mainly to References 2, 4 and 6.

An important factor to consider is that testing must occur under environment similar to those in which the items must to operate.

There are two types of interrupted or truncated tests:

- **Type I**: Time-terminated life tests, whatever the number of failures has occurred; and
- **Type II:** Failure-terminated life tests, whatever the test time has elapsed.

For each type of test, there are two possibilities: with replacement or without replacement of the failed item. Therefore, there are four types of life tests.

Whatever the type of test, it can be shown, using the so-called Maximum Likelihood method, that the failure rate and **MTTF** are well estimated by the expressions:

$$\bar{\lambda} = \frac{\Delta \mathbf{n}}{\mathbf{T}} \cong \lambda \qquad (2)$$

$$\overline{\mathbf{MTTF}} = 1/\lambda = \frac{\mathbf{T}}{\Delta \mathbf{n}} \cong \mathbf{MTTF} \qquad (3)$$

where **Δn** is the number of failures that have occurred in the test and **T** is the accumulated time in hours by the units which have not failed and the units that have failed. The bar used above indicates that the parameter is an estimate

With any number of samples in the test, the Maximum Likelihood Method is a good estimator. This is important because one can not always work with many samples in a test.

The value of **Δn** is evident because it is just the number of failures. The problem is to determine **T**. This is what we will do next.

Test Type I with Replacement of Failed Units - Be **n** the number of units submitted to testing and **Δt** the time interval set for the test. The units that fail are replaced by other good units with the same part number.

Note that **n** is constant throughout the test

The accumulated time **T** by the units that have not failed and by those that have failed is given by:

$$\mathbf{T = n\Delta t} \qquad (4)$$

Demonstrate this by using a numerical example. Consider, for example, a test with a test time **Δt** and 10 units (n = 10). Suppose that there are three failures in the instants $\mathbf{t_1}$, $\mathbf{t_2}$ and $\mathbf{t_3}$, respectively.

Note also that the total number of units submitted for testing is **n + Δn**

Test Type I without Replacing of Failed Units - In this case, if **n** units are submitted to the test in a time interval **Δt** and **Δn** units fail in that interval, the total time of hours accumulated by the survivors and failed units is given by:

$$\mathbf{T} = \sum_{\mathbf{i=1}}^{\mathbf{i=\Delta n}} \mathbf{T_i} + (\mathbf{n} - \Delta\mathbf{n})\, \Delta\mathbf{t} \qquad (5)$$

where $\sum_{\mathbf{i=1}}^{\mathbf{i=\Delta n}} \mathbf{T_i}$ is the time accumulated by the failed units units, and **(n - Δn).Δt** is the time accumulated by units that have not failed.

Test Type II with Replacement of Failed Units - In this case, n units are subjected to the test and it is established a number of failures **q**. The failed units are replaced by other good units with the same part number, except the **q-th** unit failed.

Suppose that the **q-th** unit fails at the time $\mathbf{T_q}$**.** Therefore, the time **T** accumulated by the failed units and by the not failed units is given by:

$$\mathbf{T = nT_q} \qquad (6)$$

The total number of units under test is given by:

$$\mathbf{n + q - 1} \qquad (7)$$

Test Type II without replacement of Units Failed - In this type of test, failed units are not replaced by new ones, and the time test **Δt** is fixed.

When the q-th expected failure occurs, the test is stopped in some instant **Δt**. The time **T** accumulated by the surviving units and the failed units is given by:

$$\mathbf{T} = \sum_{\mathbf{i-1}}^{\mathbf{i=q}} \mathbf{T_i} + (\mathbf{n-q})\mathbf{T_q} \qquad (8)$$

Where $\sum_{\mathbf{i-1}}^{\mathbf{i=q}} \mathbf{T_i}$ is the time accumulated by the _ the failed units and $\mathbf{(n\text{-}q)T_q}$ is the cumulative time of the units which have not failed.

Note that the total number of units used in the test is **n**, since no failed unit has been replaced.

Finally, we propose the following exercise, using the four types of tests, in the sequence shown.

Ten resistors are subjected to testing, ending 900 hours. Eight resistors fail before completing the 900 hours. Determine **T**, the failure rate $\bar{\lambda}$ and the $\overline{\mathbf{MTTF}}$, considering the following situations:

(a) the failed components are replaced;
(b) the failed components are not replaced.
(c) repeat (a) and (b), assuming that the test is terminated when seven resistors fail.

The time to failure of the units, in hours, are:
190, 295, 406, 421, 540, 670, 695 and 726.

Resp.:

(a) $\bar{\lambda}$ = 8,9 x $10^{-4}h^{-1}$ e $\overline{\text{MTTF}}$ = 1.123,6h;
(b) $\bar{\lambda}$ = 1,4 x $10^{-3}h^{-1}$ e $\overline{\text{MTTF}}$ = 717,9h;
(c) $\bar{\lambda}$ = 1,0 x $10^{-3}h^{-1}$ e $\overline{\text{MTTF}}$ = 1.000h; e
(d) $\bar{\lambda}$ = 1,3 x $10^{-3}h^{-1}$ e $\overline{\text{MTTF}}$ = 757,4h.

Note that although we have the same test data, the type of test and the effect of replacing of failed units can influence in the parameter estimatives.

Which one would you use in Assessing the parameters of one electronic equipment that will operate or which Already be operating in a fleet of airplanes? Justify.

References

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